Investigation of mixed-mode dynamic fracture in syntactic foams using digital image correlation method and high-speed photography

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Abstract
In this work, a combination of digital image correlation and high-speed photography is used to study mixed-mode dynamic fracture in syntactic foams under stress wave loading conditions. Edge cracked foam samples are subjected to mixed mode loading by impacting eccentrically relative to the initial crack plane in three point bend configuration. Ultra high-speed CCD digital camera is used to record the spray painted random speckle patterns on the specimen surface around the crack tip at framing rates of 200,000 frames per second. Two sets of images are recorded, one set before impact and another set after impact. Entire crack tip deformation history, from the time of impact up to complete fracture is mapped. A regularized restoration smoothing technique, which smoothes the displacements while allowing for discontinuity of displacements across the crack faces is developed. Over-deterministic least-squares analyses of crack opening and sliding displacement fields are used to obtain mixed-mode dynamic stress intensity factor histories for both pre- and post-crack initiation periods. The stress intensity factor histories obtained from the image correlation method are compared with the ones from computations.

Introduction
The focus of the current work is fracture behavior of syntactic foams. These are light-weight particulate composites manufactured by dispersing prefabricated hollow microballoons in a matrix material. The porosity in these materials results in lower density and superior thermal, dielectric, fire resistant, hygroscopic properties and sometimes radar or sonar transparency. They can also be tailored to suit a particular application by selecting microballoons from a wide range of materials (glass, carbon or polymer microballoons) to be used with different matrix materials (metal, polymer or ceramic).

The measurement of real-time surface deformations during a dynamic failure event such as fracture initiation and propagation is quite challenging due to a combination of spatial and temporal resolution demands involved. One of the very early efforts in this regard dates back to the work of de Graaf [1]. In this paper, photoelastic measurement was attempted to witness stress waves around a dynamically growing crack in steel. This method continues to be a popular choice when dealing with opaque solids [2, 3]. In recent years, Coherent Gradient Sensing (CGS) has become a powerful interferometric tool for studying mode-I as well as mixed-mode dynamic fracture of opaque solids because of its robustness and insensitivity to rigid body motion and vibrations [4-7]. Moiré interferometry has also been used to measure in-plane crack tip displacement fields in dynamic fracture experiments [8]. The interferometric techniques, however, generally involve elaborate surface preparation (transferring of gratings in case of moiré interferometry and preparing a specularly reflective surface in case of CGS, birefringent coatings in reflection photoelasticity, etc.). For cellular materials (syntactic foams, polymer metal foams, cellulosic materials, etc.) such surface preparations are rather challenging and in some cases may not be feasible. In such instances, digital image correlation could be a very useful tool due to the relative simplicity of surface preparation. It involves decorating a surface with black and white paint mists alternatively. Recent advances in image processing methodologies and ubiquitous computational capabilities have made it possible to apply this technique to a variety of applications - bio-mechanics, metal forming, characterization of C/C composites, - to name a few.

With the advent of digital high-speed cameras in recent years, recording rates as high as several millions frames per second at a relatively high spatial resolution are possible. This has opened the possibility of using DIC to estimate surface displacements and strains for estimating fracture/damage parameters. In the current work, the DIC technique is extended to mixed-mode dynamic fracture studies by estimating stress intensity factors for a stationary and a propagating crack tip in edge cracked syntactic foam sheets subjected to stress wave loading. A rotating mirror type high-speed digital camera is used to record random speckle pattern in the crack tip vicinity. The entire crack tip deformation and dominant strain history from the time of impact to complete fracture is mapped. Over-deterministic least-squares analyses of crack tip displacement fields are used to obtain dynamic stress intensity factors for both pre- and post-crack initiation periods. The stress intensity factor histories obtained from the image correlation method are compared with the ones from computations.

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stress intensity factor (SIF) histories for both pre- and post-crack initiation periods. The SIF histories obtained from the image correlation method are compared with the ones from finite element computations.

The DIC approach
In the digital image correlation technique, random speckle patterns on specimen surface are monitored during a fracture event. These patterns, one before and one after the deformation are acquired, digitized, and stored. Then a sub-image in the undeformed image is chosen and its location in the deformed image is sought. Once the location of a sub-image in the deformed image is found, the local displacements can be readily quantified. In the current work, a three-step approach is developed in a MATLAB™ environment to estimate 2D displacements and strains.

In the first step, 2D cross-correlation [10] is performed between two selected sub-images. The peak of the correlation function was detected to sub-pixel accuracy (1/16th of a pixel) by bicubic interpolation. This process is repeated for the entire image to get full-field in-plane displacements. In the second step, an iterative approach is used to minimize the 2D correlation coefficient by using nonlinear optimization technique. In the current work, the Newton-Raphson method [11] which uses line search and BFGS algorithm to update an inverse Hessian matrix is employed. In the third step, a regularized restoration filter [12] with a second order fit was employed for smoothing of displacements. This method uses an unbiased optimum smoothing parameter based on the noise level present in the displacement field and at the same time allows for discontinuity of displacements across the crack faces, thus preserving the strain concentration near the crack tip.

Experimental set-up
A schematic of the experimental set-up used in this study is shown in Fig. 1. It consisted of Instron-Dynatup 9250-HV drop-tower for impacting the specimen and a Cordin 550 ultra-high-speed digital camera for capturing the images in real-time. The drop-tower had an instrumented tup for recording the impact force history and a pair of anvils for recording support reaction histories. The set-up also consisted of a delay/pulse generator to generate a trigger pulse when the tup contacts the specimen. Since all the images were recorded during the dynamic event lasting over a hundred micro seconds, the set-up used two high-energy flash lamps, triggered by the camera, to illuminate the specimen. The set-up also utilized two computers, one to record the tup force and anvil reaction histories (5 MHz acquisition rate) and the other to record the images.

Figure 1: Schematic of the dynamic experimental set-up

The high-speed camera uses a combination of CCD based imaging technology and high-speed rotating mirror optical system. It can capture images up to 2 million frames per second at a resolution of 1K x 1K pixels per
image. It has 32 independent CCD image sensors positioned radially around a rotating mirror which sweeps light over these sensors. Each sensor is illuminated by a separate optical relay. Thus small misalignments between images are to be expected. These misalignments preclude the possibility of image correlation between two images recorded by different CCD sensors. Hence, an alternative approach was adopted. Prior to impacting the specimen, a set of 32 images of the specimen were recorded at the desired framing rate (200,000 frames per second in this work). While keeping all the camera settings (CCD gain, flash lamp duration, framing rate, trigger delay, etc.) same, next set of images, this time triggered by the impact event, were captured. For every image in the deformed set, there is a corresponding image in the undeformed set. That is, if an image in the deformed set was recorded by sensor #10, then the image recorded by the same sensor #10 in the undeformed set was chosen for image correlation. In order to get meaningful results, it is essential that no extraneous camera movements occur while recording a set of images and during the time-interval between the two sets of images. This was achieved by triggering the camera electronically.

Sample preparation
Edge cracked syntactic foam samples were prepared for conducting mixed-mode dynamic fracture experiments. These samples were made by mixing 25% (by volume) of hollow microballoons in low-viscosity epoxy matrix. The microballoons used in this study were commercially available hollow glass spheres of mean diameter of ∼ 60 µm and wall thickness ∼ 600 nm. The elastic modulus and Poisson’s ratio of the cured material measured ultrasonically were 3.02 GPa and 0.34 respectively [13]. Before casting the epoxy resin-hardener mixture, a sharp razor blade was inserted into the mold. When the sample was cured and removed from the mold, an edge ‘crack’ was left behind in the specimen. Further details about this method can be found in Ref. [14]. Finally, the specimen was machined into a beam of height 50 mm with a crack of 10 mm length (a/W = 0.2) as shown in Fig. 2(a). Subsequently, a random speckle pattern was created on the specimen surface by spray-painting with black and white paints alternatively.

Experimental procedure
The sequence of events in a typical experiment was as follows: The specimen was initially rested on two instrumented supports/anvils. The camera, anchored firmly to the ground, was focused on 31x31 mm² region of the sample in the crack tip vicinity (see Fig. 2(a)). A set of 32 pictures of the stationary sample were recorded at 200,000 frames per second and stored. Next, an impactor was launched at a velocity of 4.5 m/sec towards the sample. As soon as the tup made contact with a adhesive backed copper tape affixed to the top of the specimen, a trigger signal was generated by pulse/delay generator and was fed into the camera. The camera sent a separate trigger signal to the high intensity flash lamps. A trigger delay was pre-set in the camera system to capture images 85 µs after the initial impact. This time delay provides sufficient time for the high intensity flash lamps to ramp up to their full intensity levels and provide uniform illumination during recording. Since measurable deformations around the crack tip for the first 85 µs are relatively small, there

![Figure 2: (a) Specimen configuration for mixed mode dynamic fracture experiment, (b) Impactor force history and support reaction histories recorded by the drop tower and (c) finite element mesh used for elasto-dynamic simulations.](image-url)
was no significant loss of information during this period. A total of 32 images were recorded with 5 µs interval between images for a total duration of 160 µs. Once the experiment was complete, the recorded images were stored in the computer. Just before the impact occurs, the velocity of the tup was recorded by the drop-tower system. Also recorded were tup force and the support reaction histories. These are shown in Fig. 2(b). In this plot, the tup making multiple contacts with the specimen as evidenced by more than one peak can be seen. The crack initiation in this experiment occurred at about 175 µs and the specimen failed at about 240 µs. Therefore only the first peak of the impact force history is of relevance here. The impact force is recorded by both left and right anvils. Since left support is closer to the impact point, impact force records start earlier than for the right support. Also, it should be noted that anvils register noticeable impact force after 220 µs by which time the crack has propagated through half of the sample width. Therefore the reactions from the anvils do not play any role in the fracture of the sample up to this point. Accordingly, the sample was subsequently modeled as a free-free beam in finite element simulations with specimen inertia resisting the impact forces.

Finite element simulations

Elasto-dynamic finite element simulations of the current problem were conducted up to crack initiation under plane stress conditions. The finite element mesh used is shown in Fig. 2(c) along with the force boundary conditions at the impact point. Experimentally determined material properties (elastic modulus = 3.1 GPa, Poisson’s ratio 0.34 and mass density = 870 kg/m³) were used as inputs for finite element analysis. The numerical model was loaded using the force history recorded by the instrumented tup. (Before applying, the force history data was interpolated and smoothed for the following two reasons: (a) The time step of the force history measurement was larger than the one used in the simulations and (b) The force history recorded by the tup had experimental noise. Therefore smoothed cubic splines were fitted to the force history data before applying to the model.) The implicit time integration scheme of the Newmark β method with parameters β = 0.25 and γ = 0.5 and 0.5% damping was adopted in the simulations. The details of finite element analysis are avoided here and can be found elsewhere [14]. The simulation results were used to obtain instantaneous stress intensity factors up to crack initiation. The mode-I and mode-II stress intensity factors (SIF) were calculated by regression analysis of crack opening and sliding displacements, respectively.

Results

From each experiment 64 images were available, 32 from the undeformed set and 32 from the deformed set, each having a resolution of 1000 x1000 pixels. Figure 3 shows two selected speckle pattern images from the deformed set of 32 images. The time instant at which the images were recorded after impact is indicated below each image and the current crack tip is denoted by an arrow. In the current experiment, crack initiated at about 175 µs. Upon initiation, crack rapidly accelerated and subsequently propagated at a relatively steady velocity of ~270 m/s. The magnification used in this experiment was such that the size of a pixel was equal to 31 µm on the specimen. A sub-image size of 26 x 26 pixels was chosen for image correlation. The in-plane displacements were estimated for all the 32 image-pairs. The crack opening displacement, v, and sliding displacement, u, for two sample images (one before crack initiation and one after) are shown in Fig. 4. Figures 4(a) and (c) show v- and u-displacements at 150 µs after impact and Figs. 4(b) and (d) show the corresponding displacement components at t = 220 µs after impact. These are smoothed values of displacements. A significant amount of rigid body displacement component can be seen in the u-field (Figs. 4(c) and (d)). In the current work, the displacements were resolved to an accuracy of 2 to 4% of a pixel or 0.6 to 1.2 µm.
Extraction of stress intensity factors: Both crack opening and sliding displacement fields were used to extract dynamic stress intensity factors in the current work. The asymptotic expressions for crack tip displacement fields for a for a steadily propagating crack are [15],

\[
\begin{align*}
\frac{u_x}{2\mu} & = \sum_{n=1}^{\infty} \left( K_i \right) B_i (C) \sqrt{\frac{2}{\pi}} (n+1) \left\{ r_1^{n+1/2} \cos \frac{n}{2} \theta_1 - h(n) r_2^{n+1/2} \cos \frac{n}{2} \theta_2 \right\}, \\
\frac{u_y}{2\mu} & = \sum_{n=1}^{\infty} \left( K_i \right) B_i (C) \sqrt{\frac{2}{\pi}} (n+1) \left\{ -\beta_1 r_1^{n+1/2} \sin \frac{n}{2} \theta_1 + \frac{h(n)}{\beta_2} r_2^{n+1/2} \sin \frac{n}{2} \theta_2 \right\}, \\
\end{align*}
\]

where

Figure 4: Crack opening and sliding displacements (in \( \mu m \)) for pre- and post-crack initiation instants. (a) \( v \)-displacement and (c) \( u \)-displacement before crack initiation (at \( t=150\ \mu s \)); (b) \( v \)-displacement and (d) \( u \)-displacement after crack initiation (\( t=220\ \mu s \)). Crack initiation time \( \sim 175\ \mu s \). (A large rigid body displacement can be seen in (c) and (d) due to movement of the sample.)
\[ r_n = \sqrt{X^2 + \beta_n^2 \nu^2}, \quad \theta_n = \tan^{-1}\left(\frac{\beta_n Y}{X}\right) \quad m = 1, 2, \quad \beta_1 = \sqrt{1 - \left(\frac{c}{C_1}\right)^2}, \quad \beta_2 = \sqrt{1 - \left(\frac{c}{C_2}\right)^2} \]

\[ C_z = \frac{(\kappa + 1) \mu}{(\kappa - 1) \rho}, \quad C_s = \sqrt{\frac{\mu}{\rho}}, \quad \kappa = \frac{3 - \nu}{1 + \nu}, \quad h(n) = \begin{cases} \frac{2\beta_1\beta_2}{1 + \beta_1^2} & \text{for odd } n \\ \frac{1 + \beta_1^2}{2} & \text{for even } n \end{cases} \]

\[ B_i(c) = \frac{1 + \beta_1^2}{D}, \quad B_e(c) = \frac{2\beta_2}{D}, \quad D = 4\beta_1\beta_2 - \left(1 + \beta_1^2\right)^2. \]

In the above equations, \( u_x (\equiv u) \) and \( u_y (\equiv v) \) are crack sliding and opening displacements. Also, \((X, Y)\) and \((r, \theta)\) are crack the tip Cartesian and polar coordinates instantaneously aligned with the current crack tip (see, Fig. 4(b)) and \( c \) is the speed of the propagating crack tip, \( C_z \) and \( C_s \) are dilatational and shear wave speeds in the material, \( \mu \) and \( \nu \) are shear modulus and Poisson's ratio, respectively. The coefficients \((K_1), (K_2)\) of the leading terms \((n = 1)\) are the mode-I and mode-II dynamic stress intensity factors, respectively. The parameter \( \kappa \) is \((3 - \nu)/(1 + \nu)\) for plane stress conditions. Further, the above equations reduce to the form of a dynamically loaded stationary crack in the limit crack speed \( c \to 0 \). The resulting equations also implicitly assume that inertial effects enter the coefficients while retaining the functional form of the quasi-static crack tip equation.

For mode-I problem \( u_x (\equiv u) \) is the dominant in-plane displacement and is used for extracting mode-I SIF history. However, in a mixed-mode problem, both \( u_x \) and \( u_y \) displacements can be present. It can be thought that crack opening displacement \( u_y \) as having mode-I rich information while as sliding displacement \( u_x \) as having mode-II rich information. Therefore, in this work, \( u_y \) displacement is used to extract the mode-I SIF \( K_I \) and \( u_x \) to extract the mode-II SIF \( K_{II} \). For extracting SIF from displacement data, the current crack tip location was identified and the Cartesian and polar coordinate systems \((X - Y)\) and \((r - \theta)\) were established at the crack tip. A number of data points (typically 100 to 120) were collected in the region around the crack tip \((0.3 < r/B < 1.6)\) and \(-145^\circ < \theta < 145^\circ\), where \( B \) is sample thickness and \( u_x \) and \( u_y \) displacement values as well the location of these points are stored. An over-deterministic least-squares analysis [16] of the data was carried out in order to find \( K_I \) and \( K_{II} \). This was repeated for all the 32 image pairs and the stress intensity factor histories were generated.

Figure 5 shows SIF histories extracted from displacements. The crack initiation time is indicated by a vertical dotted line. Both mode-I and mode-II SIFs increase monotonically up to crack initiation at \(\sim 175 \mu s\). At crack initiation there is a noticeable drop in the magnitude of both \( K_I \) and \( K_{II} \) due to elastic unloading near the crack tip. Following initiation at \(\sim 1.0 \text{ MPam}^{1/2} \), \( K_I \) continues to increase until it reaches a value of \(\sim 1.6 \text{ MPam}^{1/2}\) beyond which it shows a decreasing trend whereas the Mode-II SIF, \( K_{II} \), remains close to zero. The SIF histories
evaluated from experiments are in good agreement with the ones from finite element computations up to crack initiation.

**Conclusions**
The DIC technique combined with high-speed digital imaging is successfully developed to study mixed-mode dynamic fracture of syntactic foams. The entire crack tip deformation history from the time of impact up to complete fracture of the specimen is mapped. The stress intensity factor histories are extracted from the displacements obtained from DIC. The SIF histories obtained from experiments are in good agreement with the ones from finite element computations up to crack initiation. In the present work, the displacements were resolved to an accuracy of 2 to 4% of a pixel (0.6 to 1.2 µm). The current approach is a powerful method to investigate dynamic failure events in real time.

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**References**